21

MA1DIP401

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination, July/August 2003

Common to All Branches

Advanced Mathematics - II

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks

1. (a) Find the angle between two lines whose direction Cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) **(6 Marks)**

   (b) Find the angle between any two diagonals of a cube. **(7 Marks)**

   (c) Find the coordinates of the foot of the perpendicular from A(1,1,1) to the line Joining b(1, 4, 6) and C(5, 4, 4). **(7 Marks)**

2. (a) Derive the equation to the plane in the intercept from $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ **(6 Marks)**

   (b) Find the equation of the plane which passes through the point $(3,-3,1)$ and is normal to the line joining the points $(3,2,-1)$ and $(2,-1,5)$. **(7 Marks)**

   (c) Find the angle between the planes $x-y+z-6=0$ *and* $2x+3y+z+5=0$. **(7 Marks)**

3. (a) Determine the value of a so that $\vec{A}=2\hat{l}+a\hat{J}+\hat{k}$ *and* $\vec{B}=4\hat{i}-2\hat{J}-2\hat{k}$ are perpendicular. **(6 Marks)**

   (b) Prove that $[\vec{A}+\vec{B},\ \vec{B}+\vec{C},\ \vec{C}+\vec{A}]=2[\vec{A},\vec{B},\vec{C}]$ **(7 Marks)**

   (c) Find the constant a so that the vectors $2\hat{l}-\hat{J}+\hat{k},\ \hat{l}+2\hat{J}-3\hat{k}$ *and* $3\hat{l}+a\hat{J}+5\hat{k}$ are coplanar. **(7 Marks)**

4. (a) If $\vec{r}=sinl\hat{i}+Cost\hat{j}+t\hat{k},\ find\ \frac{d\vec{r}}{dt},\ \frac{d^2\vec{r}}{dt^2},\ |\frac{d\vec{r}}{dt}|\ and\ |\frac{d^2\vec{r}}{dt^2}|$. **(6 Marks)**

   (b) If $\vec{A}=\vec{A}(t)$ *and* $\vec{B}=\vec{B}(t)$, where t is a scalar variable, Prove that $\frac{d}{dt}\left(\vec{A}\cdot\vec{B}\right)=\vec{A}\cdot\frac{d\vec{B}}{dt}+\frac{d\vec{A}}{dt}\cdot\vec{B}$ **(7 Marks)**

   (c) A particle moves so that its position vector is given by $\vec{r}=Coswt\ \hat{l}+sinwt\hat{J}$. Where ω is a constant. Show that

   i) the velocity $\vec{V}$ of the particle is perpendicular to $\vec{r}$

   ii) the acceleration $\vec{a}$ is directed towards the origin. **(7 Marks)**

5. (a) Find a unit normal vector to the surface $x^2y+2xz=4$ at the point (2, −2, 3 **(6 Marks)**

   (b) If $\vec{F}=(x+y+1)\hat{l}+\hat{J}-(x+y)\hat{k}$, show that $\vec{F}\ .Curl\ \vec{F}=0$ **(7 Marks)**

   (c) Find the constants a,b,c so that the vector $\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{J}+(4x+cy+2z)\hat{k}$ is irrotational. **(7 Marks)**

Contd.... 2

6. (a) Find the Laplace transform of $f(t) = e^{at}$ **(5 Marks)**

(b) Find i) $L\left[e^{-t}\, cos^2\, t\right]$ ii) $L\left[\frac{1-e^{at}}{t}\right]$ **(5+5 Marks)**

(c) Find $L[t\ sinat]$ **(5 Marks)**

7. (a) If $L[f(t)] = F(s)$, then show that $L\left[\int_0^t f(t)dt\right] = \frac{1}{s}F(s)$ **(5 Marks)**

(b) Find i) $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$ ii) $L^{-1}\left[\frac{4s+5}{(s-1)^2(s+2)}\right]$ **(5+5 Marks)**

(c) Find $L^{-1}\left[log\ \frac{s+a}{s+b}\right]$ **(5 Marks)**

8. (a) Using Laplace transforms, solve

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^{-t} \text{ given that } y = 2,\ \frac{dy}{dt} = -1\ at\ t = 0$$

(10 Marks)

(b) Solve the simultaneous equations using Laplace transforms :

$$\frac{dx}{dt} + y = sint \quad \frac{dy}{dt} + x = Cost \text{ given that } x = 2, y = 0\ for\ t = 0$$

(10 Marks)

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MATDIP401

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination, July/August 2004

Common to All Branches

Advanced Mathematics - II

Time: 3 hrs.] [Max.Marks : 100

Note: ***1. Answer any FIVE full questions.***
2. All questions carry equal marks

1. (a) Find the angle between any two diagonals of a cube. **(6 Marks)**

   (b) With usual notation derive the equation of a plane in the form
   $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ **(7 Marks)**

   (c) Find the equation of the plane through the points $(1,-2,2),(-3,1,-2)$ and perpendicular to the plane $2x-y-z+6=0$ **(7 Marks)**

2. (a) Find the equations of a straight line perpendicular to both the lines
   $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z+2}{3}$ and $\frac{x+2}{2}=\frac{y-5}{-1}=\frac{z+3}{2}$
   and passing through their point of intersection. **(6 Marks)**

   (b) Find the reflection of the point $(1,-2,3)$ in the plane $2x+3y+2z+3=0$ **(7 Marks)**

   (c) Find the magnitude and the equations of the line of shortest distance between the lines
   $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ **(7 Marks)**

3. (a) Find a vector of magnitude 12 units which is perpendicular to the vectors $4i-j+3k$ and $-2i+j-2k$ **(6 Marks)**

   (b) Given $\vec{a}=2i+3j-k$, $\vec{b}=i-2k-j$ and $\vec{c}=-i+2j+2k$, find
   i) $[\vec{a},\vec{b},\vec{c}]$ ii) $(\vec{a}\times\vec{b})\times\vec{c}$ **(7 Marks)**

   (c) Find the value of λ so that the points $A(-1,4,-3)$, $B(3,2,-5)$, $C(-3,8,-5)$ and $D(-3,\lambda,1)$ may lie in one plane. **(7 Marks)**

4. (a) If $\frac{d\vec{a}}{dt}=\vec{w}\times\vec{a}$, $\frac{d\vec{b}}{dt}=\vec{w}\times\vec{b}$, show that
   $\frac{d}{dt}(\vec{a}\times\vec{b})=\vec{w}\times(\vec{a}\times\vec{b})$ **(6 Marks)**

   (b) A particle moves along a curve $x=cos2t$, $y=sin2t$, $z=t$. Find the velocity and accelaration at $t=\frac{\pi}{8}$ along $\sqrt{2}i+\sqrt{2}j+k$ **(7 Marks)**

   (c) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x=z^2+y^2-3$ at the point $(2,-1,2)$ **(7 Marks)**

5. (a) Prove that $Curl(\phi\vec{A})=\phi(Curl\vec{A}+(grad\phi)\times\vec{A}$ **(6 Marks)**

   (b) If $\vec{F}=(x+y+1)i+j-(x+y)k$ show that $\vec{F}.curl\vec{F}=0$ **(7 Marks)**

   (c) Prove that $\nabla^2(r^n)=n(n+1)r^{n-2}$ where $r=|xi+yj+zk|$ **(7 Marks)**

Contd.... 2

6. (a) If $L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$ show that $L\{t^n\} = \frac{n!}{s^{n+1}}$ **(5 Marks)**

(b) Find i) $L\{e^{3t+7} + 4sin^2 3t + 2cos4t cos2t\}$

ii) $L\{\frac{cos2t - cos3t}{t}\}$ **(5+5 Marks)**

(c) Show that $\int_0^\infty te^{-3t} sint\ dt = \frac{3}{50}$ **(5 Marks)**

7. (a) If $L\{f(t)\} = F(s)$, show that $L\{\int_0^t f(t)dt\} = \frac{1}{s}F(s)$ **(5 Marks)**

(b) Find the inverse laplace transforms of the following functions:

i) $\frac{s}{(s+2)(s^2+1)}$

ii) $log\{\frac{s^2+1}{s(s-1)}\}$

iii) $\frac{s+1}{s^2-s+1}$ **(5+5+5 Marks)**

8. (a) Using Laplace transform method solve

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^{2t} \text{ with } y(0) = 0 \text{ and } y'(0) = 1$$ **(10 Marks)**

(b) Solve the following differential equations using laplace transform method.

$$\frac{dx}{dt} + \frac{dy}{dt} + x = -e^{-t}$$

$$\frac{dx}{dt} + 2\frac{dy}{dt} + 2x + 2y = 0$$

given that $x(0) = -1,\ y(0) = 1$ **(10 Marks)**

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MATDIP401

USN

Fourth Semester B.E Degree Examination, July/August 2004

Common to All Branches

Advanced Mathematics - II

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks

1. (a) Find the angle between any two diagonals of a cube. (6 Marks)

(b) With usual notation derive the equation of a plane in the form
$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (7 Marks)

(c) Find the equation of the plane through the points $(1,-2,2),(-3,1,-2)$ and perpendicular to the plane $2x-y-z+6=0$ (7 Marks)

2. (a) Find the equations of a straight line perpendicular to both the lines
$\frac{x-1}{1}=\frac{y-1}{2}=\frac{z+2}{3}$ and $\frac{x+2}{2}=\frac{y-5}{-1}=\frac{z+3}{2}$
and passing through their point of intersection. (6 Marks)

(b) Find the reflection of the point $(1,-2,3)$ in the plane $2x+3y+2z+3=0$ (7 Marks)

(c) Find the magnitude and the equations of the line of shortest distance between the lines
$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ (7 Marks)

3. (a) Find a vector of magnitude 12 units which is perpendicular to the vectors $4i-j+3k$ and $-2i+j-2k$ (6 Marks)

(b) Given $\vec{a}=2i+3j-k$, $\vec{b}=i-2k-j$ and $\vec{c}=-i+2j+2k$, find
i) $[\vec{a},\vec{b},\vec{c}]$ ii) $(\vec{a}\times\vec{b})\times\vec{c}$ (7 Marks)

(c) Find the value of λ so that the points $A(-1,4,-3)$, $B(3,2,-5)$, $C(-3,8,-5)$ and $D(-3,\lambda,1)$ may lie in one plane. (7 Marks)

4. (a) If $\frac{d\vec{a}}{dt}=\vec{w}\times\vec{a}$, $\frac{d\vec{b}}{dt}=\vec{w}\times\vec{b}$, show that
$\frac{d}{dt}(\vec{a}\times\vec{b})=\vec{w}\times(\vec{a}\times\vec{b})$ (6 Marks)

(b) A particle moves along a curve $x=cos2t$, $y=sin2t$, $z=t$. Find the velocity and accelaration at $t=\frac{\pi}{8}$ along $\sqrt{2}i+\sqrt{2}j+k$ (7 Marks)

(c) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x=z^2+y^2-3$ at the point $(2,-1,2)$ (7 Marks)

5. (a) Prove that $Curl(\phi\vec{A})=\phi(Curl\vec{A}+(grad\phi)\times\vec{A}$ (6 Marks)

(b) If $\vec{F}=(x+y+1)i+j-(x+y)k$ show that $\vec{F}.curl\vec{F}=0$ (7 Marks)

(c) Prove that $\nabla^2(r^n)=n(n+1)r^{n-2}$ where $r=|xi+yj+zk|$ (7 Marks)

Contd.... 2

6. (a) If $L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$ show that $L\{t^n\} = \frac{n!}{s^{n+1}}$ **(5 Marks)**

(b) Find i) $L\{e^{3t+7} + 4sin^2 3t + 2cos4tcos2t\}$

ii) $L\{\frac{cos2t-cos3t}{t}\}$ **(5+5 Marks)**

(c) Show that $\int_0^{\infty} te^{-3t} sint\ dt = \frac{3}{50}$ **(5 Marks)**

7. (a) If $L\{f(t)\} = F(s)$, show that $L\{\int_0^t f(t)dt\} = \frac{1}{s}F(s)$ **(5 Marks)**

(b) Find the inverse laplace transforms of the following functions:

i) $\frac{s}{(s+2)(s^2+1)}$

ii) $log\{\frac{s^2+1}{s(s-1)}\}$

iii) $\frac{s+1}{s^2-s+1}$ **(5+5+5 Marks)**

8. (a) Using Laplace transform method solve

$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^{2t}$ with $y(0) = 0$ and $y'(0) = 1$ **(10 Marks)**

(b) Solve the following differential equations using laplace transform method.

$\frac{dx}{dt} + \frac{dy}{dt} + x = -e^{-t}$

$\frac{dx}{dt} + 2\frac{dy}{dt} + 2x + 2y = 0$

given that $x(0) = -1,\ y(0) = 1$ **(10 Marks)**

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MATDIP401

USN

Fourth Semester B.E Degree Examination, January/February 2004

Common to All Branches

Advanced Mathematics - II

Time: 3 hrs.] 22 [Max.Marks : 100

Note: 1. *Answer any FIVE full questions.*

1. (a) The direction cosines l,m,n of two lines are connected by the relations
$l+m+n=0, 2lm+2ln-mn=0$
Find them. **(8 Marks)**

(b) Define a plane. Find the equation of the plane in the normal form in the usual notation. **(6 Marks)**

(c) Find the equation of the plane through the points $(2,2,1),(1,-2,3)$ and parallel to the x-axis. **(6 Marks)**

2. (a) Find the symmetrical form the equations of the line
$x+y+z-1=0=2x-y-3z+1$ **(6 Marks)**

(b) Find the image of the point (2, −3,4) with respect to the plane $4x+2y-4z+3=0$ **(8 Marks)**

(c) Find the equations of the two straight lines through the origin each of which intersects the straight line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ and is inclined at an angle of 60^0 to it. **(6 Marks)**

3. (a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices P, Q, R of a triangle, show that the vector area of the triangle PQR is
$\frac{1}{2}(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})$ **(6 Marks)**

(b) Prove that $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}(\vec{c}\cdot\vec{a})-\vec{c}(\vec{a}\cdot\vec{b})$ **(8 Marks)**

(c) Show that the four points whose position vectors are
$3\vec{i}-2\vec{j}+4\vec{k}$, $6\vec{i}+3\vec{j}+\vec{k}$,
$5\vec{i}+7\vec{j}+3\vec{k}$ and $2\vec{i}+2\vec{j}+6\vec{k}$
are coplanar. **(6 Marks)**

4. (a) A particle moves along the curve
$x=t^3+1,\quad y=t^2,\quad z=2t+5,$
where 't' is the time. Find the components of its velocity and accelaration at time $t=1$ in the direction $2\vec{i}+3\vec{j}+6\vec{k}$ **(7 Marks)**

(b) The temperature at any point in space is given by $T=xy+yz+zx$. Determine the derivative of T in the direction of the vector $3\vec{i}-4\vec{k}$ at $(1,1,1)$ **(6 Marks)**

(c) If ϕ is a scalar field and $\vec{A}$ is a vector field, prove that $\text{curl}(\phi\vec{A})=\phi(\text{curl}\vec{A})+(\text{grad}\phi)\times\vec{A}$ **(7 Marks)**

5. (a) Find the values of the constants a,b,c for which the vector
$\vec{V}=(x+y+az)\vec{i}+(bx+3y-z)\vec{j}+(3x+cy+z)\vec{k}$ is irrotational. **(6 Marks)**

(b) If ϕ is a scalar field, prove that $\text{curl}(\text{grad}\phi)=0$ **(7 Marks)**

Contd.... 2

(c) Prove that $\frac{\vec{r}}{|\vec{r}|^3}$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ is both solenoidal and irrotational. **(7 Marks)**

6. (a) Find i) $L(\cos at)$ ii) $L(\sin h\ at)$ **(8 Marks)**

(b) Find i) $L(e^{-3t} \cos t \sin 2t)$

ii) $L(\frac{\sin at}{t})$

iii) $L(t^3 \cos t)$

iv) $L \int_0^t f(t)\, dt$ **(12 Marks)**

7. (a) Express the following function in terms of the unit step function and hence find the Laplace transform.

$$f(t) = \begin{cases} t^2, & 1 < t \leq 2 \\ 4t, & t > 2 \end{cases}$$ **(8 Marks)**

(b) Find i) $L^{-1}\left[\frac{4s+5}{(s-1)^2(s+2)}\right]$

ii) $L^{-1}\left[\frac{1}{(s-1)(s^2+1)}\right]$

iii) $L^{-1}\left(\log(\frac{s+a}{s+b})\right)$ **(12 Marks)**

8. (a) Prove that

$$Lf''(t) = s^2 Lf(t) - sf'(0) - f''(0)$$ **(6 Marks)**

(b) By employing the convolution theorem, find

$$L^{-1}\frac{s}{(s^2+a^2)^2}$$ **(6 Marks)**

(c) Solve the equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$$

under the conditions $y(0) = 1, \quad y'(0) = 0$ **(8 Marks)**

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MATDIP401

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination, February/March 2005

Common to All Branches

(Old Scheme)

Advanced Mathematics - II

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*

1. (a) If $A(4,3,2)$, $B(5,4,6)$, $C(-1, 1,5)$ are the corners of a triangle find the co-ordinates of the point in which the bisector of the angle A meets the side BC. **(6 Marks)**

   (b) Find the ratio in which X O Y plane divides the line joining of the points (-3, 4, 8) and (5, -6, 4) and thus write the co-ordinates of the point of division. **(7 Marks)**

   (c) Prove that the condition that the lines whose direction cosines are $l, m, n \& l', m', n'$ should be

   i) Perpendicular if $ll' + mm' + nn' = 0$

   ii) Parallel if $l = l',\ m = m', n = n'$. **(7 Marks)**

2. (a) Find the equation of the plane which passes through the point $(3, -3, 1)$ and is parallel to the plane $2x + 3y + 5z + 6 = 0$. **(6 Marks)**

   (b) Find the distance of the point $(1, 2, 3)$ from the plane, $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ **(7 Marks)**

   (c) Obtain the equation of the plane passing through the line of intersections of the planes $7x - 4y + 7z + 16 = 0 \& 4x + 3y - 2z + 13 = 0$ and perpendicular to the plane $x - y - 2z + 5 = 0$ **(7 Marks)**

3. (a) Find div F and cur F when

$$F = (3x^2 - 3yz)i + (3y^2 - 3xz)j + (3z^2 - 3xy)k$$ **(6 Marks)**

   (b) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$

   where $r = |\vec{r}| = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ **(7 Marks)**

   (c) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is time. Find the velocity and acceleration at any time t. Also find the initial velocity & initial accelerations. **(7 Marks)**

Contd.... 2

4. (a) Find the directional derivative of the function $\phi = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point (1, 1, 1). (6 Marks)

(b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$. (7 Marks)

(c) Show that $\vec{F} = \frac{xi+yj}{x^2+y^2}$ is both solenoidal and irrotational. (7 Marks)

5. (a) Prove that (6 Marks)

$$\text{curl (curl F)} = \text{grad div } F - \nabla^2 F$$

(b) If V_1 and V_2 be the vectors joining fixed points (x_1, y_1, z_1) &(x_2, y_2, z_2) respectively to a variable point then prove that $div\ (V_1 \times V_2) = 0$. (7 Marks)

(c) Find the unit tangent vector at any point on the curve $x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$. (7 Marks)

6. (a) Prove that : $L\{\ sin\ at\ \} = \frac{a}{s^2+a^2}$ (5 Marks)

(b) Find

i) $L\{e^{-3t}\ sin\ 2t\}$

ii) $L\{\frac{t}{2a}\ sinhat\}$.

iii) $L\{\frac{1-e^t}{t}\}$

(15 Marks)

7. (a) If $L\{f(t)\} = J(s)$, then

$$L\{\tfrac{1}{t}\ f(t)\} = \int_s^\infty J\ (s)\ ds$$

provided the integral exits (5 Marks)

(b) Find

i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$

ii) $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$

iii) $L^{-1}\{\ log(\frac{s+1}{s})\}$

(15 Marks)

8. (a) Solve the following using Laplace transform method $y'' - 3y' + 2y = 12e^{-t}$, given that $y(0) = 2,\ y'(0) = 6$. (10 Marks)

(b) Using Laplace transform method solve the following simultaneous equations.

$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$$

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$$

given $x = 0 = y\ at\ t = 0$ (10 Marks)

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MATDIP401

USN | | | | | | | | | |

Fourth Semester B.E Degree Examination, February/March 2005

Common to All Branches

Advanced Mathematics - II

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks.

1. (a) Find the angle between any two diagonals of a cube. **(7 Marks)**

(b) If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction cosines of two lines subtending an angle θ between them then prove that $cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. **(7 Marks)**

(c) Compute the angle between the lines whose direction ratios are 2, 1, 1 and $4, (\sqrt{3}-1), (-\sqrt{3}-1)$. **(6 Marks)**

2. (a) Derive the equation of the plane in the normal form $lx + my + nz = p$. **(7 Marks)**

(b) Find the equation of the plane passing through the point (1, 2, -1) and perpendicular to the planes $x + y - 2z = 5$ and $3x - y + 4z = 12$. **(7 Marks)**

(c) Find the image of the point $(1, -1, 2)$ in the plane $2x + 2y + z = 11$. **(6 Marks)**

3. (a) Find the value of the constant λ so that the vectors $\vec{A} = i - j + 2k$, $\vec{B} = 2i + j - 3k$ and $\vec{C} = \lambda i - j + k$ **(7 Marks)**

(b) For three vectors $\vec{A}, \vec{B}, \vec{C}$, prove that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$ **(7 Marks)**

(c) If $\vec{A} = 4i + 3j + k$ and $\vec{B} = 2i - j + 2k$, find a unit vector N perpendicular to the vectors $\vec{A}$ and $\vec{B}$ such that $\vec{A}$, $\vec{B}, N$ form a right handed system. **(6 Marks)**

4. (a) At any point of the curve $x = 3cost$, $y = 3sint$, $Z = 4t$ find i) Tangent vector and unit tangent vector at $t = 0$ **(7 Marks)**

(b) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the velocity and the acceleration of the particle at the time $t = 1$. **(7 Marks)**

(c) If $\frac{d\vec{A}}{dt} = \vec{\omega} \times \vec{A}$ and $\frac{d\vec{B}}{dt} = \vec{\omega} \times \vec{B}$, then prove that $\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{\omega} \times (\vec{A} \times \vec{B})$ **(6 Marks)**

5. (a) Find a unit normal vector to the surface $x^2 + 3y^2 - 2z^2 = 6$ at the point $p(2, 0, 1)$ **(7 Marks)**

(b) Prove that for every field $\vec{V}$, $div(curl\ \vec{V}) = 0$ where $\vec{V} = V_1 i + V_2 j + V_3 k$ **(7 Marks)**

(c) Compute the values of the constants a, b, c such that the vector $\vec{V} = (x + y + az)i + (bx + 3y - z)j + (3x + cy + z)k$ is irrotational. **(6 Marks)**

6. (a) If $f(t) = \begin{cases} 2t \ for \ 0 < t < 5 \\ 1 \ for \ t > 5 \end{cases}$, find $L\{f(t)\}$. **(5 Marks)**

Contd.... 2

(b) Find i) $L\left\{cos^3 2t\right\}$

ii) $L\left\{\frac{sin^2 t}{t}\right\}$

iii) $L\left\{e^{-3t} sin^2 2t\right\}$ **(15 Marks)**

7. (a) If $L\left\{f(t)\right\} = \int_0^\infty e^{-st} f(t) dt$, then prove that

$L\left\{f''(t)\right\} = s^2 L\left\{f(t)\right\} - sf(0) - f'(0)$ **(5 Marks)**

(b) Find i) $L^{-1}\left\{\frac{1}{s^2-5s+6}\right\}$

ii) $L^{-1}\left\{\frac{s-1}{s^2+25-6s}\right\}$

iii) $L^{-1}\left\{log\left(1+\frac{\omega^2}{s^2}\right)\right\}$ **(15 Marks)**

8. (a) Using Laplace transforms, solve $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t$ with $y(0) = 2,\ y'(0) = -1$. **(10 Marks)**

(b) Using Laplace transforms, solve the simultaneous differential equations given below.

$\frac{dx}{dt} - y = e^t$

$\frac{dy}{dt} + x = sint$

where $x(0) = 1$ and $y(0) = 0$ **(10 Marks)**

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MATDIP401

USN

Fourth Semester B.E Degree Examination, July/August 2005

Common to All Branches

(Old Scheme)

Advanced Mathematics - II

Time: 3 hrs.] [Max.Marks : 100

Note: ***1. Answer any FIVE full questions.***
2. All questions carry equal marks

1. (a) Define the direction cosines of a straight line. If l,m,n are the direction cosines of a straight line, Prove that $l^2 + m^2 + n^2 = 1$ **(6 Marks)**

(b) If θ is the angle between two lines whose direction cosines are (l_1, m_1, n_1) *and* (l_2, m_2, n_2), prove that $cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ **(7 Marks)**

(c) Find the reflection of the point (2,-1,3) in the plane 3x-2y-z = 1 **(7 Marks)**

2. (a) Derive the equation of the plane in the intercept form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ **(6 Marks)**

(b) Find the equation of the plane through the line of intersection of the planes x+y+z = 1 and 2x+3y +4z = 5 and perpendicular to the plane x-y+z = 0 **(7 Marks)**

(c) Find the angle between the planes 2x-y+z = 1 and x+y+2z = 3 **(7 Marks)**

3. (a) Find a unit vector normal to the plane of

$\vec{a} = 3i - 2J + 4K, \quad \vec{b} = i + J - 2K$

Find also the sine of the angle between them. **(6 Marks)**

(b) If $\vec{A} = i - 2J - 3K, \quad \vec{B} = 2i + J - K, \quad \vec{C} = i + 3J - K$

Find $(\vec{A} \times \vec{B}) \times (\vec{B} \times \vec{C})$ **(7 Marks)**

(c) Show that $\vec{A} = i - 2J + 3K, \quad \vec{B} = 2i + J + K, \quad \vec{C} = 3i + 4J - K$ are coplanar. **(7 Marks)**

4. (a) If $\vec{A} = 5t^2 i + tJ - t^3 K, \quad \vec{B} = sin\ t\ i - cos\ t\ J$

Find: (i) $\frac{d}{dt}(\vec{A} \cdot \vec{B})$

(ii) $\frac{d}{dt}(\vec{A} \times \vec{B})$ **(6 Marks)**

(b) A particle moves along the curve $x = t^3 - 4t, y = t^2 + 4t, z = 8t^2 - 3t^3$, where t denotes time. Find the magnitude of velocity and acceleration in the direction of the vector 2i+2J-K (7 Marks)

(c) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the vector i+2J+2K (7 Marks)

5. (a) Prove that $\nabla \cdot \nabla \times \vec{F} = 0$ (6 Marks)

(b) Find the divergence and curl of the vector

$\vec{F} = (xyz + y^2 z)i + (3x^2 y + y^2 z)J + (xz^2 - y^2 z)K$ (7 Marks)

(c) Find the constants a,b, so that the vector

$\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)K$ is irrotatoinal (7 Marks)

6. (a) Find the laplace transform of $f(t) = e^{at}$ (5 Marks)

(b) Find (i) $L\{sin^3 2t\}$

(ii) $L\left\{\frac{1-\cos t}{t}\right\}$ (5+5=10 Marks)

(c) Find $L\{t \sin at\}$ (5 Marks)

7. (a) Find $L\{f(t)\} = F(s)$ then show that

$$L\left\{\int_o^t f(u)du\right\} = \frac{1}{s}F(s)$$ (5 Marks)

(b) Find :

$$L^{-1}\left\{\frac{s+3}{s^2-4s+13}\right\}$$

$$L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$$

$$L^{-1}\left\{\frac{s+1}{s-1}\right\}$$ (5+5+5=15 Marks)

8. (a) Using Laplace transform method solve :

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t} \text{ with}$$

$$y(o) = y^1(o) = 1$$ (10 Marks)

(b) Solve the following differential equations using loplace transform method

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0$$

Given that x = y = 0 when t = 0 (10 Marks)

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MATDIP401

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Fourth Semester B.E. Degree Examination, Dec.08 / Jan.09
Advanced Mathematics II

3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. Show that the lines whose direction cosines are given by the equations $l+m+n=0$, $al^2+bm^2+cn^2=0$ are perpendicular if a+b+c = 0. **(06 Marks)**

b. Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$. **(07 Marks)**

c. If P, Q, A, B are (1, 2, 3), (-2, 1, 3), (4, 4, 2), (2, 1, -4), find the projection of PQ on AB. **(07 Marks)**

2 a. Find the equation of the plane in the intercept form. **(06 Marks)**

b. Find the equation of the plane which passes through (3, -3, 1) and is perpendicular to the planes $7x+y+2z=6$ and $3x+5y-6z=8$. **(07 Marks)**

c. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$, $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar. Find their common point. **(07 Marks)**

3 a. Show that the four points whose position vectors are $3i-2j+4k$, $6i+3j+k$, $5i+7j+3k$ and $2i+2j+6k$ are coplanar. **(06 Marks)**

b. A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+5$ where t is the time. Find the components of its velocity and acceleration at t = 1 in the direction $2i+3j+6k$. **(07 Marks)**

c. If $\vec{A}=4i+3j+k$, $\vec{B}=2i-j+2k$ find a unit vector N perpendicular to vectors A and B. Such that A, B, N form a right-handed system. **(07 Marks)**

4 a. Find the angle between the tangents to the curve $\vec{r}=t^2i+2tj-t^3k$ at the point $t=\pm1$. **(06 Marks)**

b. Let $\vec{a}=i+j-k$, $\vec{b}=i-j+k$, $\vec{c}=i-j-k$. Find the vector $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$. **(07 Marks)**

c. Find a unit vector normal to the surface $x^2+3y^2+2z^2=6$ at (2, 0, 1). **(07 Marks)**

5 a. Find the directional derivative of $f(x,y,z)=xy^2+yz^3$ at the point (2, -1, 1) in the direction of i+2j+2k. **(06 Marks)**

b. Find i) $\text{div}\left(3x^2i+5xy^2j+xyz^3k\right)$ at (1, 2, 3).

ii) $\text{curl}\left[xyzi+3x^2yj+\left(xz^2-y^2z\right)k\right]$ **(07 Marks)**

c. Find the values of the constants a, b, c for which the vector $v=(x+y+az)i+(bx+3y-z)j+(3x+cy+z)k$ is irrotational. **(07 Marks)**

6 a. Find the Laplace transform of

$$f(t)=\begin{cases} e^t; 0<t<1 \\ 0; \quad t>1 \end{cases}.$$ **(05 Marks)**

b. Find $L\left\{e^{-3t}(2\cos 5t-3\sin 5t)\right\}$. **(05 Marks)**

c. Evaluate $L\left\{t\sin^2 t\right\}$. **(05 Marks)**

d. Find $L\left\{\frac{1-e^t}{t}\right\}$. **(05 Marks)**

7 Find the **inverse** Laplace transform for the following:

a. $\dfrac{s^2-3s+4}{s^3}$. (05 Marks)

b. $\dfrac{s+2}{s^2-4s+13}$. (05 Marks)

c. $\dfrac{s^2+s-2}{s(s+3)(s-2)}$. (05 Marks)

d. $\log\left(\dfrac{s+a}{s+b}\right)$. (05 Marks)

8 a. Use Laplace transform method to solve,
$\dfrac{d^2x}{dt^2}-2\dfrac{dx}{dt}+x=e^t$ with $x=2$, $\dfrac{dx}{dt}=-1$ at $t=0$. (10 Marks)

b. Solve the following simultaneous equations using Laplace transform method,
$\dfrac{dx}{dt}-y=e^t$; $\dfrac{dy}{dt}+x=\sin t$; given $x(0)=1$, $y(0)=0$. (10 Marks)

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MATDIP401

Fourth Semester B.E. Degree Examination, June / July 08
Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. Obtain an expression for the angle between two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2). Hence obtain the condition for the lines to be perpendicular and parallel. **(07 Marks)**

b. Prove that the lines whose direction cosines are given by the relations $l + m + n = 0$ and $al^2 + bm^2 + cn^2 = 0$ are i) perpendicular, if $a + b + c = 0$, and ii) parallel, if $\left(\frac{1}{a}\right)+\left(\frac{1}{b}\right)+\left(\frac{1}{c}\right)=0$. **(07 Marks)**

c. For what value of λ, are the four points $(0, -1, \lambda)$, $(2, 1, -1)$, $(1, 1, 1)$, $(3, 3, 0)$ coplanar? Find the equation of the plane through them. **(06 Marks)**

2 a. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and perpendicular to the plane $2y - 3z = 4$. **(07 Marks)**

b. Find the image (reflection) of the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{4}$ in the plane $2x + y + z = 6$. **(07 Marks)**

c. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$; $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar, find their common point and the equation of the plane in which they lie. **(06 Marks)**

3 a. Determine λ and μ by using vectors, such that the points $(-1, 3, 2)$ $(-4, 2, -2)$ and $(5, \lambda, \mu)$ lie on a straight line. **(07 Marks)**

b. If four points whose position vectors are $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar, show that $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{a}\,\vec{b}\,\vec{d}] + [\vec{a}\,\vec{d}\,\vec{c}] + [\vec{a}\,\vec{b}\,\vec{c}]$. **(07 Marks)**

c. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove that $[\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$. Hence find $\vec{a}\times(\vec{b}\times\vec{c})$ for $\vec{a} = \vec{i} + \vec{j} - \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \vec{k}$, $\vec{c} = \vec{i} - \vec{j} - \vec{k}$. **(06 Marks)**

4 a. The position vector of a particle at time t is $\vec{r} = \cos(t-1)\vec{i} + \sinh(t-1)\vec{j} + \alpha t^3\vec{k}$. Find the condition imposed on α by requiring that at time $t = 1$, the acceleration is normal to the position vector. **(07 Marks)**

b. In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^2 y^2 z^4$ maximum? Find also the magnitude of this maximum. **(07 Marks)**

c. Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ for $n \neq -1$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. **(06 Marks)**

5 a. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $r = |\vec{r}|$ and $\vec{a}$ is a constant vector, find the value of $\operatorname{div}\left(\frac{\vec{a}\times\vec{r}}{r^n}\right)$. **(07 Marks)**

b. Find the constant 'k_1' so that $\vec{A} = y(k_1x^2 + z)\vec{i} + x(y^2 - z^2)\vec{j} + 2xy(z - xy)\vec{k}$ is solenoidal. **(07 Marks)**

c. Show that $\vec{F}\cdot\operatorname{curl}\vec{F} = 0$ for $\vec{F} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$. **(06 Marks)**

6 a. Prove that $L[t^n] = \frac{n!}{S^{n+1}}$ when n = 0, 1, 2, 3, - - - - -. (07 Mar

b. Find the Laplace transform of
i) $e^{-t}\cos^3 3t$ ii) $t\,e^{2t}\sin^2 2t$. (07 Marks)

c. Find the Laplace transform of
$$\int_0^t \frac{e^t \sin t}{t}\,dt.$$ (06 Marks)

7 a. Prove that :
$$L^{-1}\left[\frac{s}{s^4+4a^4}\right] = \frac{1}{2a^2}\sin at \sin h\, at.$$ (07 Marks)

b. Find :
$$L^{-1}\left[\frac{s}{\left(s^2+a^2\right)^2}\right].$$ (07 Mar

c. Find
$$L^{-1}\left[\cot^{-1}\frac{s}{2}\right].$$ (06 Marks)

8 a. Solve the following differential equation by Laplace transform method :
$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y = e^{-t}\sin t, \text{ where } y(0)=0 \text{ and } y'(0)=1.$$ (10 Marks)

b. Using Laplace transform method solve
$$\frac{dx}{dt}+5x-2y = t,\ \frac{dy}{dt}+2x+y = 0, \text{ being given } x = y = 0 \text{ when } t = 0$$ (10 Marks)

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Fourth Semester B.E. Degree Examination, June-July 2009

Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions***

1 a. If l, m, n and l', m', n' are the direction cosines of the lines OP & OQ and θ be the angle between them then show that $\cos\theta = ll' + mm' + nn'$. Also derive the condition for the perpendicularity of OP & OQ. **(07 Marks)**

b. Find the equation of the plane which passes through the point (3, –3, 1) and is
   i) Parallel to the plane $2x + 3y + 5z + 6 = 0$.
   ii) Perpendicular to the planes $7x + y + 2z = 6$ and $3x + 5y - 6z = 8$. **(07 Marks)**

c. Find the equations to the two planes which bisects the angles between the planes $3x - 4y + 5z = 3$ and $5x + 3y - 4z = 9$. **(06 Marks)**

2 a. Obtain the equation of a plane passing through the line of intersection of the planes $7x - 4y + 7z + 16 = 0$ and $4x + 3y - 2z + 13 = 0$ and perpendicular to the plane $x - y - 2z + 5 = 0$. **(07 Marks)**

b. Find the equation of the two straight lines through the origin each of which intersects the straight line $\frac{1}{2}(x - 3) = y - 3 = z$ and it is inclined at an angle of 60° to it. **(07 Marks)**

c. Find the magnitude and the equation of the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$. **(06 Marks)**

3 a. If $\vec{A} = i + 2j + 3k$, $\vec{B} = -i + 2j + k$, $\vec{C} = 3i + j$ Find P such that $\overrightarrow{A + PB}$ is perpendicular to $\vec{C}$. Also, find the dot product of A with B and $A\cdot(B + C)$. **(07 Marks)**

b. If $\vec{A} = 4i + 3j + k$, $\vec{B} = 2i - j + 2k$ find a unit vector perpendicular to the plane containing both $\vec{A}$ & $\vec{B}$. Also, show that A is not perpendicular to $\vec{B}$. **(07 Marks)**

c. Find the constant a such that the vectors $\overrightarrow{AB} = ai - 5j + 2k$, $\overrightarrow{AD} = -7i + 14j - 3k$, $\overrightarrow{AC} = 11i + 4j + k$ are coplanar. **(06 Marks)**

4 a. A particle moves along a curve $x = e^{-t}$, $y = \cos 3t$, $z = 2\sin 3t$ where t is the time variable. Determine its velocity and acceleration vectors and also, find the magnitudes of velocity and acceleration at $t = 0$. **(07 Marks)**

b. Find the directional derivative of $f(xyz) = xy^2 + yz^3$ at the point (2, –1, 1) in the direction of the vector I + 2J + 2K. Also, find the direction along which it is maximum. **(07 Marks)**

c. Prove that $\nabla r^n = 2r^{n-2} R$ where $R = xi + yj + zk$ and $r = |R|$. **(06 Marks)**

5 a. For what value of 'a' does this vector $\vec{P} = (ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{K}$ has zero divergence. Also find $\nabla \times \vec{P}$. **(07 Marks)**

b. Show that curl gra $\phi = \vec{0}$ **(07 Marks)**

c. Given that $\vec{F} = (x + y + 1)i + j - (x + y)k$. Show that $\vec{F}\cdot \text{curl}\, F = 0$ **(06 Marks)**

6 a. Using the definition show that $L\{t^n\} = \frac{n!}{s^{n+1}}$ **(05 Marks)**

b. Find $L\{e^{-t}\, t \cos at\}$ **(05 Marks)**

c. Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$ **(05 Marks)**

d. Find $L\{\cos(at + b)\}$ **(05 Marks)**

7 a. Show that $L\{t^n\} = (-1)^n \frac{dn}{ds^n} f(s)$ where n is an integer. **(05 Marks)**

b. Find $L^{-1}\left\{\frac{s^2 - 10s + 13}{(s-7)(s^2 - 5s + 6)}\right\}$ **(05 Marks)**

c. Find $L^{-1}\left\{\frac{s+2}{s^2 - 4s + 13)}\right\}$ **(05 Marks)**

d. If $L\{f(t)\} = f(s)$, show that $L^{-1}\left\{\int_s^{\infty} f(s)ds\right\} = \frac{f(t)}{t}$ **(05 Marks)**

8 a. Solve the initial value problem using Laplace transforms $(D^3 - 3D^2 + 3D - 1)y = 0$ given that $y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$ **(10 Marks)**

b. Solve the simultaneous equations $x' - y = e^t$, $y' + x = \sin t$, $x(0) = 1$, $y(0) = 0$. **(10 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.09/Jan.10

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8=50, will be treated as malpractice.

1 a. If (l, m, n) be the direction cosines of a line then prove that $l^2 + m^2 + n^2 = 1$. **(06 Marks)**

b. Find the value of K if the angle between the lines with direction ratios -2, 1, -1 and 1, -K, -1 is $\frac{2\pi}{3}$. **(07 Marks)**

c. Find the projection of the line segment AB on CD, where A = (3, 4, 5), B = (4, 6, 3), C = (-1, 2, 4), D = (1, 0, 5) **(07 Marks)**

2 a. Find the angle between the planes x-y+2z = 9 and 2x+y+z = 7. **(06 Marks)**

b. Find the equation of the plane passing through the line of intersection of the planes $x + 2y - 3z - 1 = 0$ and $3x - y + 4z - 5 = 0$ and perpendicular to the plane $3x - y - 3z + 4 = 0$. **(07 Marks)**

c. Find the point of intersection of the lines, $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. **(07 Marks)**

3 a. If $\vec{A} = 2i - 3j - k$ and $\vec{B} = i + 4j - 2k$, find $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$. **(06 Marks)**

b. For any three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, prove that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{b} \cdot \vec{c}\right)\vec{a}$ **(07 Marks)**

c. Prove that the four points $4i + 5j + k$, -(j+k), (3i+9j+4k) and 4(-i+j+k) are coplanar. **(07 Marks)**

4 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$ where t is the time. Find the velocity and acceleration at t = 1. **(06 Marks)**

b. Find the unit vector normal to the surface $x^2y - 2xz + 2y^2z^4 = 10$ at (2, 1, -1). **(07 Marks)**

c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2). **(07 Marks)**

5 a. If $\vec{F} = (3x^2y - z)i + (xz^3 + y^4)j - 2x^3z^2k$ find grad(div $\vec{F}$) at (2, -1, 0). **(06 Marks)**

b. Find curl(curl $\vec{A}$) given that $\vec{A} = xyi + y^2zj + z^2yk$. **(07 Marks)**

c. Show that $\vec{F} = \frac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

6 a. Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$. **(05 Marks)**

b. Find $L(t^n)$ where n is a positive integer. **(05 Marks)**

c. Find $L[t \cos at]$. **(05 Marks)**

d. Find $L\left[\frac{\cos at - \cos bt}{t}\right]$. **(05 Marks)**

7 Find the inverse Laplace transform for the following:

a. $\frac{(s+2)^3}{s^6}$ **(05 Marks)**

b. $\frac{2s-1}{s^2+4s+29}$ **(05 Marks)**

c. $\frac{2s^2+5s-4}{s^3+s^2-2s}$ **(05 Marks)**

d. $\log\left(1-\frac{a^2}{s^2}\right)$ **(05 Marks)**

8 a. Use Laplace transform method to solve, $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=e^{-t}$; $y(0)=0,\ y'(0)=0$ **(10 Marks)**

b. Find the inverse Laplace transformation of $\frac{s^2}{(s-2)^3}$ **(10 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, May/June 2010
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Find the projection of the line AB on CD where $A = (1, 2, 3)$, $B = (-1, 0, 2)$, $C = (1, 4, 2)$, $D = (2, 0, -1)$. **(06 Marks)**

b. Find the angle between two lines whose direction cosines are given by $l + 3m + 5n = 0$ and $2mn - 6nl - 5lm = 0$. **(07 Marks)**

c. A line makes angles $\alpha, \beta, \gamma, \delta$ with diagonals of a cube. Prove that $\text{Cos}^2\alpha + \text{Cos}^2\beta + \text{Cos}^2\gamma + \text{Cos}^2\delta = \frac{4}{3}$. **(07 Marks)**

2 a. Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to $5x + y + 4z = 0$. **(06 Marks)**

b. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, -1, -1) are coplanar. Find the equation of the plane containing them. **(07 Marks)**

c. Find the equation of a straight line through (7, 2, -3) and perpendicular to each of the lines. $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x+2}{4} = \frac{y-3}{5} = \frac{z-4}{6}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 3(\sqrt{3}\,\hat{i} - \hat{j})$, $\vec{b} = 6\hat{j}$, $\vec{c} = 3(\sqrt{3}\,\hat{i} + \hat{j})$ form an isosceles triangle. **(06 Marks)**

b. A particle moves along the curve $\vec{r} = 3t^2\hat{i} + (t^3 - 4t)\hat{j} + (3t + 4)\hat{k}$. Find the components of velocity and acceleration at $t = 2$ in the direction $\hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

c. Find the angle between the normals to the surfaces $x^2y^2 = z^4$ at (1, 1, 1) and (3, 3, -3). **(07 Marks)**

4 a. Find the directional derivatives of the function $\phi = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point (1, 1, 1). **(06 Marks)**

b. Find the div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. If $\vec{v} = 2xy\,\hat{i} + 3x^2y\,\hat{j} - 3ayz\,\hat{k}$ is solenoidal at (1, 1, 1), find a. **(07 Marks)**

5 a. Find the unit normal vector to the surface $xy + x + zx = 3$ at (1, 1, 1). **(06 Marks)**

b. Find the constants 'a', 'b', 'c' such that the vector field $(\sin y + az)\,\hat{i} + (bx\cos y + z)\,\hat{j} + (x + cy)\,\hat{k}$ is irrotational. Also find the scalar field ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)

c. Prove that $\nabla^2(\log r) = \frac{1}{r^2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. (07 Marks)

6 a. Find the Laplace transform of Sin 2t Sin 3t. (05 Marks)

b. Find $L\left[\frac{(1-e^t)}{t}\right]$. (05 Marks)

c. Find $L\left[e^{-t}(3\sinh 2t - 2\cosh 3t)\right]$. (05 Marks)

d. Find the Laplace transform of $f(t) = \begin{cases} t/\lambda & \text{when } 0 < t < \lambda \\ 1 & \text{when } t > \lambda \end{cases}$. (05 Marks)

7 a. Evaluate $\int_0^\infty \frac{\sin t}{t}\,dt$ using Laplace transform. (05 Marks)

b. Find the inverse Laplace transform of $\frac{1}{(s^2+3s+2)(s+3)}$. (05 Marks)

c. Find $L^{-1}\left[\frac{s-1}{s^2-6s+25}\right]$. (05 Marks)

d. Find $L^{-1}\left[\log\left\{\frac{s^2+1}{s^2-s}\right\}\right]$. (05 Marks)

8 a. Find $L^{-1}\left[\frac{1}{s^2(s+5)}\right]$ using convolution theorem. (10 Marks)

b. Solve the differential equation $y'' + 2y' + y = 6te^{-t}$ under the condition $y(0) = 0 = y'(0)$ using Laplace transform. (10 Marks)

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